A Pragmatic Approach to Robust Gain Scheduling

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Outline

• Traditional gain scheduling design

• How things can go wrong in practice
  (or “awkward moments as a control engineer”)

• Model uncertainty & closed-loop stability

• A constructive design procedure

• Summary
Standard Gain Scheduling Procedure

Begin with a nonlinear plant $y = G(u,w)$:

- Create LPV Model
- Design Linear Control
- Gain Schedule
- Analyze Performance
The Problem

- Steps 1 and 2 (linear modeling and control) are reliably followed in industrial practice.

- However, ad hoc techniques are often applied for scheduling the control (Step 3):
  - Interpolation of controller coefficients e.g. PID tuning parameters
  - Interpolation of controller outputs
  - Etc.

- Thus nonlocal stability and performance (Step 4) are often not guaranteed in practice

Practitioners need nonlinear control design techniques!
Consider set of finite-gain $\mathcal{L}_2$ stable plants $y=G(u,w)$

$$\|y_\tau\|_{L_2} \leq \gamma_{gu}\|u_\tau\|_{L_2} + \gamma_{gw}\|w_\tau\|_{L_2} + \beta_g \quad \text{for all } \tau \in [0, \infty)$$

Control design requirements:

1. **Robust stability**: closed-loop $\mathcal{L}_2$ stable for
   - all exogenous signals $w \in \mathcal{L}_{2e}$ and
   - realistic plant-model mismatch

2. **Performance**: closed-loop performance better than $K=0$.

3. **Within designers’ capability**: can $K(y,w)$ be designed using linear control techniques?
Model Uncertainty & Closed-Loop Stability
Model Uncertainty & Closed-loop Stability

If $\Delta$ and $Q$ are both finite-gain $\mathcal{L}_2$ stable:

$$\|\Delta(u, w)\|_{\mathcal{L}_2} \leq \gamma_{\Delta u} \|u\|_{\mathcal{L}_2} + \gamma_{\Delta w} \|w\|_{\mathcal{L}_2} + \beta_{\Delta}$$

$$\|Q(e, w)\|_{\mathcal{L}_2} \leq \gamma_{qe} \|e\|_{\mathcal{L}_2} + \gamma_{qw} \|w\|_{\mathcal{L}_2} + \beta_{q}$$

Then closed-loop with $G$ and $K$ is finite-gain $\mathcal{L}_2$ stable if:

$$\gamma_{\Delta u} \gamma_{qe} < 1$$

- Which may be interpreted as a tradeoff between:
  - model uncertainty $\gamma_{\Delta u}$ and
  - controller aggressiveness $\gamma_{qe}$
Gain Scheduling with Stability Guarantees: A Constructive Design Procedure
Design procedure: 1. Create LPV model

- Consider a simple plant: \( \dot{x}(t) = a(\theta)x(t) + b(\theta)u(t) \)
  \( y(t) = x(t) \)

In practice, designer may not know the underlying nonlinear structure.
Design procedure: 1. Create LPV model

- Partition space and assign local linear models:

$$\hat{G}_1(s) = \frac{-2}{s + 5} \quad \hat{G}_2(s) = 0 \quad \hat{G}_3(s) = \frac{3}{s + 7}$$

(best approx for a gain sign change)
Design procedure: 2. Design Linear Control

- Design linear controllers for each model:

\[ \hat{G}_1(s) = \frac{-2}{s + 5} \quad \hat{G}_2(s) = 0 \quad \hat{G}_3(s) = \frac{3}{s + 7} \]

\[ K_1(s) = P_1 + \frac{I_1}{s} \quad K_2(s) = 0 \quad K_3(s) = P_3 + \frac{I_3}{s} \]

(PID and other familiar structures are popular in industrial practice.)

Question: how should we switch between controllers?
Design procedure: 3. Gain Schedule

• Form the linear Youla-Kucera parameters for each of the modes $j$:

$$Q_j(s) = K_j(s)\left(1 + \hat{G}_j(s)K_j(s)\right)^{-1}$$

• And construct the gain scheduled controller:

Uses only simple components:

• banks of linear $Q_j(s)$ and $\hat{G}_j(s)$
• signal switches $\sigma(t)$
Design procedure: 4. Analyze Performance

Robust stability

- Given finite-gain $\mathcal{L}_2$ stable plant $y = G(u,w)$
- For any selection of stable linear plant models $\hat{G}_j(s)$ and switching strategy $\sigma(t)$
- It is always possible to design corresponding linear controllers $K_j(s)$
- Such that $K(y,w)$ is finite-gain $\mathcal{L}_2$ stabilizing for $G(u,w)$

Outline of Proof

$\Delta$ is finite-gain $\mathcal{L}_2$ stable with $\gamma_{\Delta u} < \infty$

Q is finite-gain $\mathcal{L}_2$ stable with

$$\gamma_{qe} \leq \sqrt{\sum_{j=1}^{m} \left\| \mathcal{Q}_j(s) \right\|_{H\infty}^2}$$

Thus it is always possible to make $\gamma_{\Delta u} \gamma_{qe} < 1$
Design procedure: 4. Analyze Performance

Closed-loop performance
- Given robustly stable closed-loop \( (\gamma_{u}\gamma_{qe} < 1) \)
- \textbf{True} closed-loop performance in a given mode:

\[
\begin{align*}
\mathbf{r} & \rightarrow K_j(s) \rightarrow \mathbf{G}(u,w) \rightarrow \mathbf{y}_p \\
& \text{is related to } \textbf{nominal} \text{ performance} \\
\end{align*}
\]

By the relation:

\[
\|y_{nom} - y_p\|_{L_2} \leq \left\|(1 + \hat{G}_j(s)K_j(s))^{-1}\right\|_{H_{\infty}}.
\]

\[
\left\{ \begin{array}{c}
\gamma_{u}\gamma_{qe} \|\mathbf{r}\|_{L_2} + \gamma_{\Delta w} \|\mathbf{w}\|_{L_2} + \frac{\beta_{\Delta}}{1 - \gamma_{u}\gamma_{qe}} \\
1 - \gamma_{u}\gamma_{qe}
\end{array} \right\}
\]

So \( y_p \rightarrow y_{nom} \) for \( \Delta \rightarrow 0 \).
Closed-loop performance

- Achievable performance depends in part on model uncertainty

- For example, integral control* in each $K_j(s)$ is possible for all modes if uncertainty is bounded by:

$$\gamma_{\Delta u} < \frac{1}{\sqrt{\sum_j 1/ \sigma(\hat{G}_j(0))^2}}$$

Roughly speaking – robust stability and integral action requires smallest steady-state singular values of local linear models should be large wrt model uncertainty.

*Integral control action in $K_j(s)$ requires setting $Q_j(0) = \hat{G}_j(0)^{-1}$
Summary

• Industry needs nonlinear control design techniques:
  - Robust stability
  - Closed-loop performance
  - Within designers’ capability

• Gain scheduling theory has challenges in bridging the gap from linear designs to nonlinear closed-loop

• We have presented a simple technique such that linear control designs can produce a finite-gain $L_2$ stable closed-loop for nonlinear plants
  - SISO and MIMO control
  - Common controller structures permitted (PID, $H_\infty$, etc)
  - Plants may include “hard” nonlinearities (e.g. actuator saturation)
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